In-medium heavy-quark spectral function: a path-integral approach

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Outline

- The physical motivations: study of medium effects on the spectral densities of HQ and $Q\overline{Q}$ correlators;
- The path-integral formulation for the problem:
  - General setting;
  - The static limit: recovering the real-time effective potential;
  - Preliminary numerical results of the MC simulations and of the spectral analysis;
  - Some physical insight: the HQ spectral function from a resummed one-loop calculation;
- Conclusions and future developments.

Our goal

We wish to perform a study resulting

- numerically less expensive then lattice calculations (hence allowing a more robust reconstruction of the spectral function);
- capable to get a deeper physical insight on the processes involved.

The basic object of our study

$$G^{>}(t) \equiv \langle \mathcal{O}(t)\, \mathcal{O}^{\dagger}(0) \rangle$$

- $\mathcal{O}^{\dagger}$ creates a Q or a $Q\overline{Q}$ pair;
- Spectral decomposition

$$\begin{aligned} G^{>}(t) &= Z^{-1} \sum_n e^{-\beta E_n} \sum_m \langle n|\mathcal{O}(t)|m\rangle\langle m|\mathcal{O}^{\dagger}(0)|n\rangle \\ &= Z^{-1} \sum_n e^{-\beta E_n} \sum_m e^{i(E_n - E_m)t} |\langle m|\mathcal{O}^{\dagger}(0)|n\rangle|^2, \end{aligned}$$

  – $G^{>}(t)$ is an **analytic function** in the strip $-\beta < \mathrm{Im} t < 0 \Longrightarrow$ *unified description of real and imaginary-time propagation*;
  – HQs: *external probe* placed *in a hot/dense medium* of light particles $\Longrightarrow \{|n\rangle\}$ do not contain heavy quarks.

Getting the in-medium spectral function...

- In the general case the spectral density of a correlator would be given by

$$\sigma(\omega) \equiv G^{>}(\omega) \mp G^{<}(\omega);$$

- Dealing with the propagation of an external probe one has $G^{<} \equiv 0$, so that

$$\sigma(\omega) = G^{>}(\omega) \quad \Longrightarrow \quad G^{>}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \sigma(\omega);$$

- The standard procedure to get $\sigma(\omega)$ is then, exploiting the analyticity of $G^{>}$:

$$\underbrace{G^{>}(t=-i\tau)}_{\text{evaluated}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \; e^{-\omega\tau} \quad \underbrace{\sigma(\omega)}_{\text{reconstructed}} \quad .$$

The general idea

Treat the heavy fermion propagating in a thermal bath as a point-like particle in Quantum-Mechanics. Hence, in evaluating the HQ euclidean correlator:

- Sum over all the possible trajectories in a given background field:

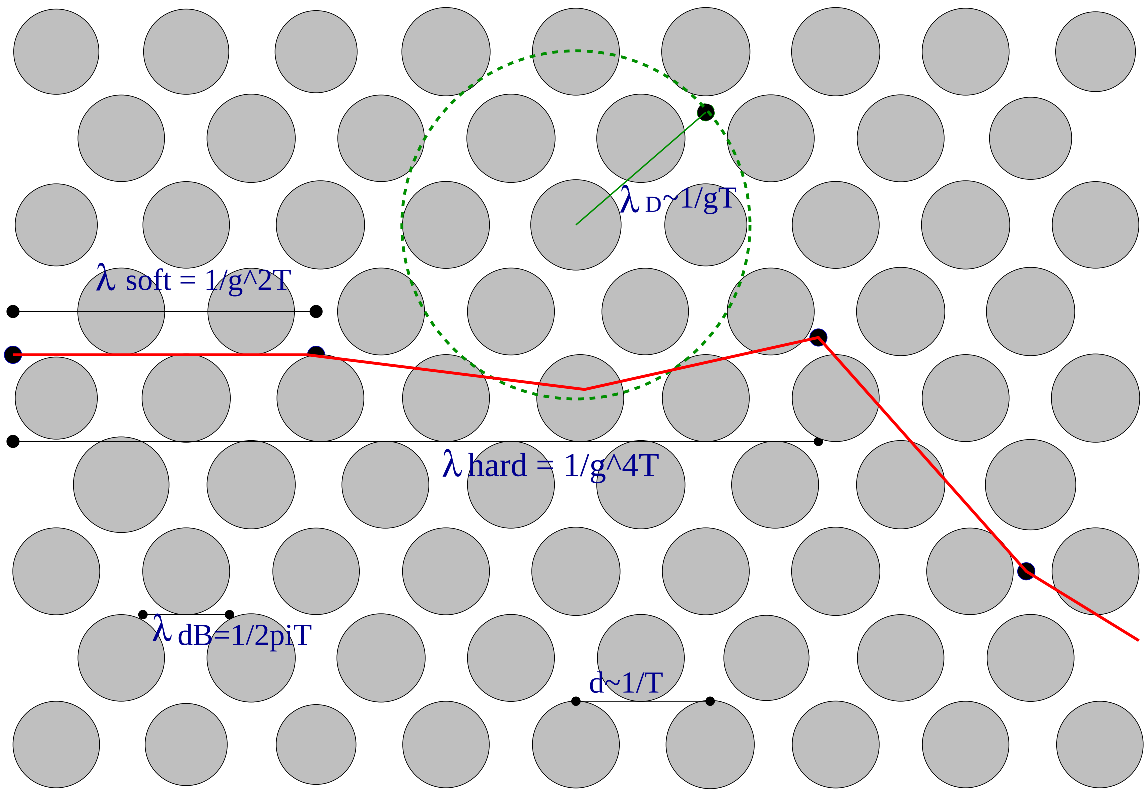
$$\langle \boldsymbol{x}_f \tau_f | \boldsymbol{x}_i \tau_i \rangle = \int_{\boldsymbol{x}(\tau_i)=\boldsymbol{x}_i}^{\boldsymbol{x}(\tau_f)=\boldsymbol{x}_f} [\mathcal{D}\boldsymbol{x}(\tau')] \exp\left[-\int_{\tau_i}^{\tau_f} d\tau' \left(\frac{1}{2} M \dot{\boldsymbol{x}}^2 + g\Phi(x)\right)\right],$$

- Average over all the possible field configurations (the action accounting for medium effects)

$$G^>(-i\tau, \boldsymbol{r}_1 | 0, \boldsymbol{r}_1') = Z^{-1} \int_{\boldsymbol{z}_1(0)=\boldsymbol{r}_1'}^{\boldsymbol{z}_1(\tau)=\boldsymbol{r}_1} [\mathcal{D}\boldsymbol{z}_1] \int [\mathcal{D}\Phi] \exp\left[-\int_0^\tau d\tau' \frac{1}{2} M \dot{\boldsymbol{z}}_1{}^2\right] \times$$
$$\times \exp\left[-g\int_0^\tau d\tau' \Phi(\tau', \boldsymbol{z}_1(\tau'))\right] e^{-S_E^{\rm eff}[\Phi]}$$

Which action to employ to weight the field configurations for a hot gauge plasma?

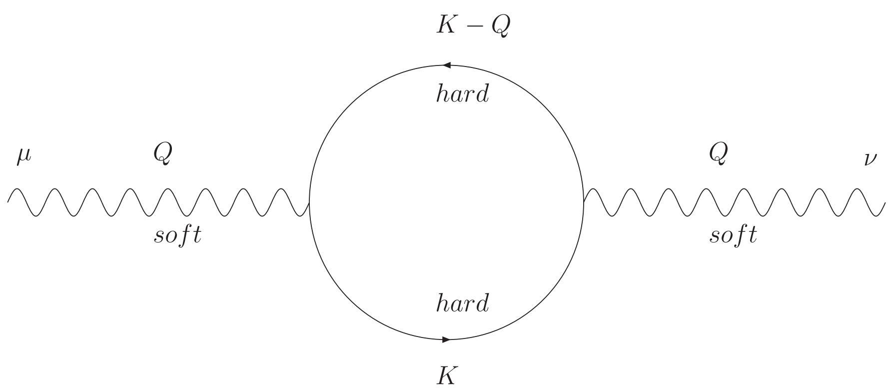
Scales in a weakly-coupled relativistic plasma



most of the scattering processes involve the exchange of soft momenta $Q \sim gT$.

The HTL effective action

- The propagation of soft (long wave-length) gauge-bosons $(Q\sim gT)$ is ***dressed by the interactions with the light plasma-particle*** which are ***hard*** $(K\sim T)$



- The HTL effective action (*for an abelian gauge plasma*):

$$S^{HTL}[A] = \frac{1}{2}\int d^4x \int d^4y\, A^\mu(x)\left(D^{-1}\right)^{HTL}_{\mu\nu}(x-y)A^\nu(y).$$

A heavy "quark" in a hot gauge plasma

Neglecting possible non-abelian effects we perform Monte Carlo simulations for

$$G^{>}(-i\tau,\boldsymbol{r}_1|0,\boldsymbol{r}'_1)=\int_{\boldsymbol{z}(0)=\boldsymbol{r}'_1}^{\boldsymbol{z}(\tau)=\boldsymbol{r}_1}[\mathcal{D}\boldsymbol{z}]\exp\left[-\int_0^\tau d\tau'\left(M+\frac{1}{2}M\dot{\boldsymbol{z}}^2\right)\right]\times$$
$$\times\exp\left[\frac{g^2}{2}\int_0^\tau d\tau'\int_0^\tau d\tau''\Delta_L^T(\tau'-\tau'',\boldsymbol{z}(\tau')-\boldsymbol{z}(\tau''))\right]$$

where

$$\begin{aligned}\Delta_L(\tau,\boldsymbol{q}) &\equiv \Delta_L^{vac}(\tau,\boldsymbol{q})+\Delta_L^T(\tau,\boldsymbol{q})\\ &= \frac{-1}{\boldsymbol{q}^2}\delta(\tau)+\int_{-\infty}^{+\infty}\frac{dq_0}{2\pi}e^{-q_0\tau}\rho_L(q_0,\boldsymbol{q})[\theta(\tau)+N(q^0)]\end{aligned}$$

is expressed in terms of the HTL spectral function

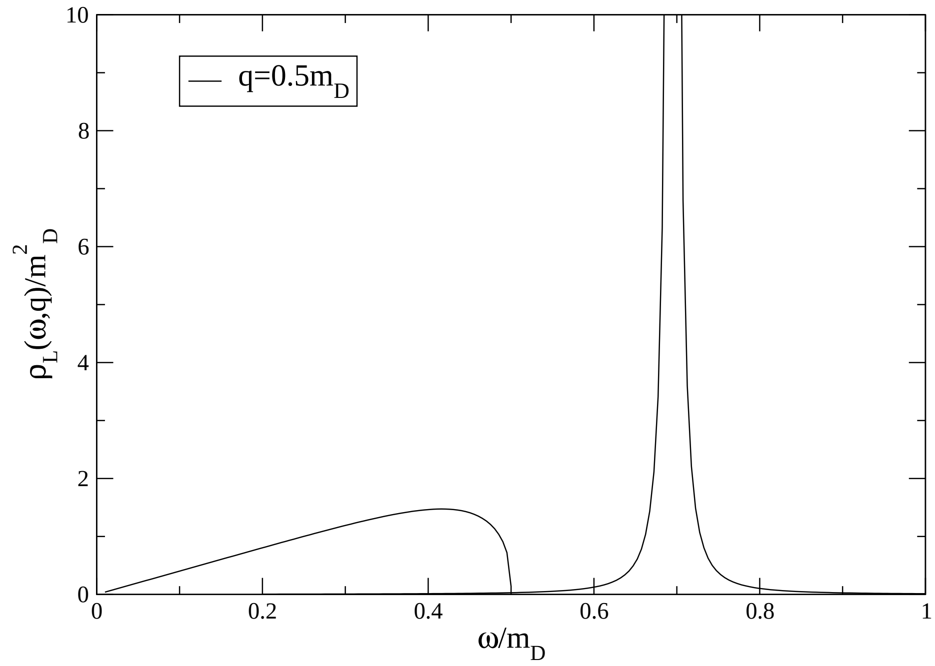
$$\rho_L(\omega>0,q)\equiv 2\pi\Big[\underbrace{Z_L(q)\delta(\omega-\omega_L(q))}_{\text{plasmon pole}}+\underbrace{\theta(q^2-\omega^2)\beta_L(\omega,q)}_{\text{Landau damping}}\Big]$$

HTL longitudinal spectral function

$$\rho_L(\omega) \equiv 2\,\mathrm{Im} D_L^{\mathrm{ret}}(\omega) = 2\,\mathrm{Im}\Delta_L(\omega + i\eta),$$

where:

$$\Delta_L(q^0, q) = \frac{-1}{q^2 + m_D^2\left(1 - \frac{q^0}{2q}\ln\frac{q^0+q}{q^0-q}\right)}$$



Pole + Continuum. The width is put by hand!

Our long term goal...

...would be to address the $Q\overline{Q}$ case within the same approach:

$$G^{>}(-i\tau;\boldsymbol{r}_1,\boldsymbol{r}_2|0;\boldsymbol{r}_1',\boldsymbol{r}_2') = e^{-(M_1+M_2)\tau}\int_{\boldsymbol{r}_1'}^{\boldsymbol{r}_1}[\mathcal{D}\boldsymbol{z}_1]\int_{\boldsymbol{r}_2'}^{\boldsymbol{r}_2}[\mathcal{D}\boldsymbol{z}_2]\times$$

$$\times\exp\left[-\int_0^\tau d\tau'\left(\frac{1}{2}M_1\dot{\boldsymbol{z}}_1{}^2-\frac{g^2}{2}\int_0^\tau d\tau''\Delta_L^T(\tau'-\tau'',\boldsymbol{z}_1(\tau')-\boldsymbol{z}_1(\tau''))\right)\right]\times$$

$$\times\exp\left[-\int_0^\tau d\tau'\left(\frac{1}{2}M_2\dot{\boldsymbol{z}}_2{}^2-\frac{g^2}{2}\int_0^\tau d\tau''\Delta_L^T(\tau'-\tau'',\boldsymbol{z}_2(\tau')-\boldsymbol{z}_2(\tau''))\right)\right]\times$$

$$\times\exp\left[-g^2\int_0^\tau d\tau'\int_0^\tau d\tau''\Delta_L(\tau'-\tau'',\boldsymbol{z}_1(\tau')-\boldsymbol{z}_2(\tau''))\right]$$

Static limit

For $M=\infty$ the HQs are frozen to their positions. The asymptotic behavior of the real-time $Q\overline{Q}$ propagator allows the to identify the in-medium effective potential:

$$\overline{G}(t,\boldsymbol{r}_1-\boldsymbol{r}_2) \underset{t\to\infty}{\sim} \exp[-iV_{\text{eff}}(\boldsymbol{r}_1-\boldsymbol{r}_2)t],$$

with

$$\underbrace{V_{\text{eff}}(\boldsymbol{r}_1-\boldsymbol{r}_2)}_{\text{effective potential}} \equiv g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)D_{00}(\omega=0,\boldsymbol{q})$$

$$= g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)\Bigg[\underbrace{\frac{1}{\boldsymbol{q}^2+m_D^2}}_{\text{screening}}-i\underbrace{\frac{\pi m_D^2 T}{|\boldsymbol{q}|(\boldsymbol{q}^2+m_D^2)^2}}_{\text{collisions}}\Bigg]$$

$$= -\frac{g^2}{4\pi}\left[m_D+\frac{e^{-m_D r}}{r}\right]-i\frac{g^2 T}{4\pi}\phi(m_D r)$$

Wide literature on the $T \neq 0$ $Q\overline{Q}$ potential

- M. Laine and collaborators: JHEP 0703:054, JHEP 0705:028, JHEP 0709:066, JHEP 0801:043.
- N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky: Phys.Rev.D78:014017,2008.
- A.B., J.P.Blaizot and C. Ratti, Nucl.Phys.A806:312,2008.
- A. Dumitru et al.: Phys.Lett.B662:37,2008; Phys.Rev.D79:054019,2009...

Numerical results

from the MC simulations for the path-integral

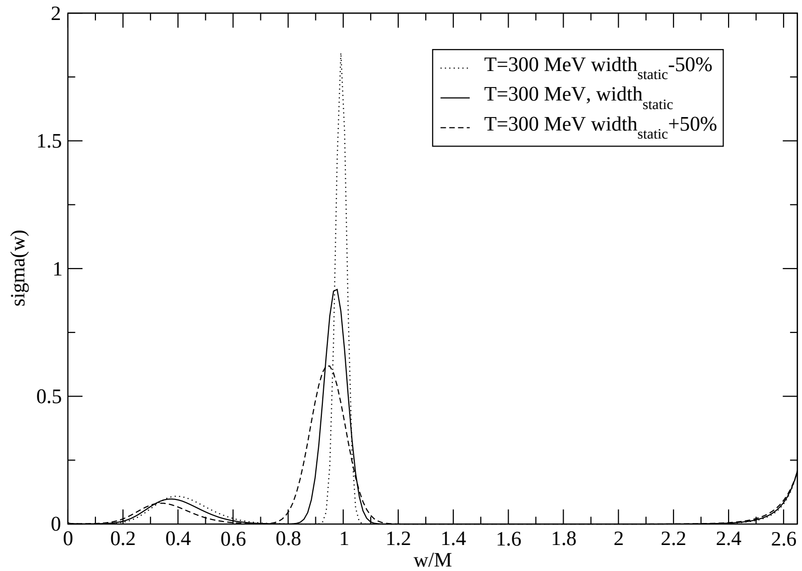
$$\underbrace{G^{>}(t\!=\!-i\tau)}_{\text{evaluated}} \equiv G(\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\; e^{-\omega\tau} \quad \underbrace{\sigma(\omega)}_{\text{reconstructed}} \quad .$$

- $G(\tau)$ *obtained after averaging over at least* 10^6 *paths!*
- The above data are used the get the HQ spectral density through a MEM analysis.

Fixing values of the parameters of possible relevance for the QGP study

$$
\begin{aligned}
\frac{g^2}{4\pi} &\equiv C_F\alpha_s, \quad with \quad \alpha_s = 0.3 \\
M &= 1.5 \text{ GeV} \ \ (\text{charm}) \\
T &= 200 - 400 \text{ MeV} \ \ (T/M \ll 1)
\end{aligned}
$$

Results for the HQ spectral function I

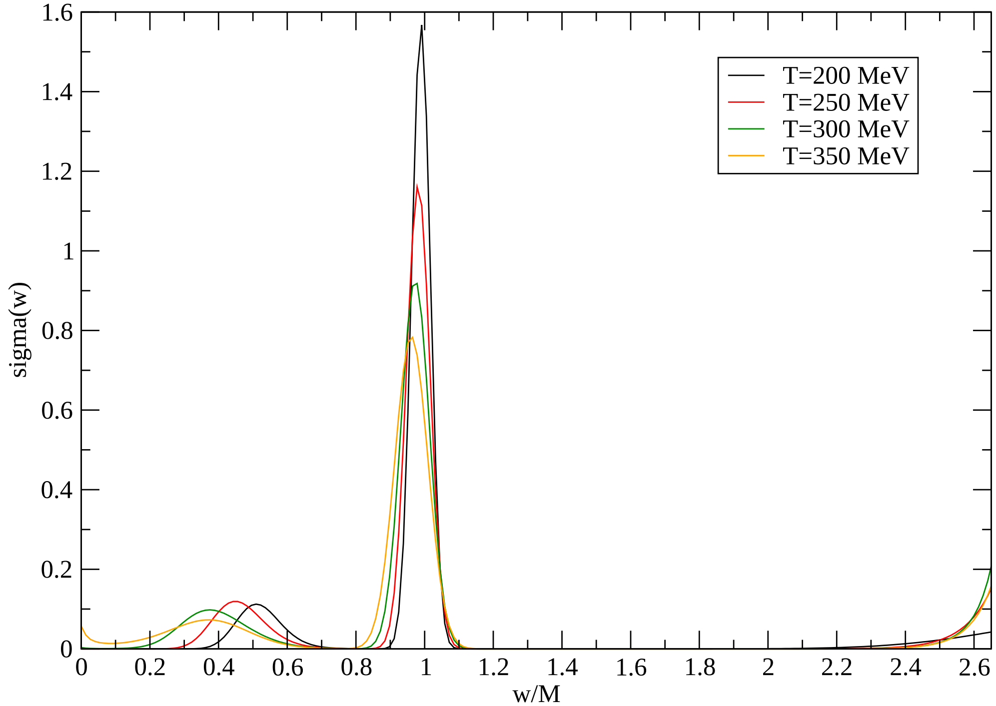


- We set $T = 300$ MeV;
- We check the sensitivity on the (gaussian) default model, satisfying:

$$\int \frac{d\omega}{2\pi}\,\sigma(\omega) = 1, \qquad \int \frac{d\omega}{2\pi}\,\omega\,\sigma(\omega) = M.$$

The appearance of a *secondary peak at low-energy* seems a robust feature of the spectral density.

Results for the HQ spectral function II



- Using a gaussian default model in the MEM...
- ...we perform a temperature scan.

As the temperature increases
the secondary peak moves toward lower energies.

In order to interpret the numerical outcomes of the simulations....

...some physical insight from (weak-coupling) **thermal field theory calculations**

General setup

- Analytic non-relativistic HQ propagator

$$G(z) = \frac{-1}{z - E_p - \Sigma(z,\boldsymbol{p})},$$

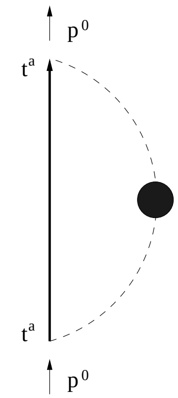
where $E_p = M + p^2/2M$ and setting $z = \omega + i\eta$ corresponds to *retarded boundary conditions*;

- HQ spectral function:

$$\sigma(\omega) \equiv 2\mathrm{Im}\, G^R(\omega) = \frac{\Gamma(\omega)}{[\omega - E_p - \mathrm{Re}\,\Sigma(\omega)]^2 + \Gamma^2(\omega)/4},$$

with $\Gamma(\omega) \equiv -2\mathrm{Im}\,\Sigma^R(\omega) \implies$ *HQ spectral function non-vanishing only for energies for which the self-energy develops an imaginary-part*.

HQ self-energy: resummed one-loop result



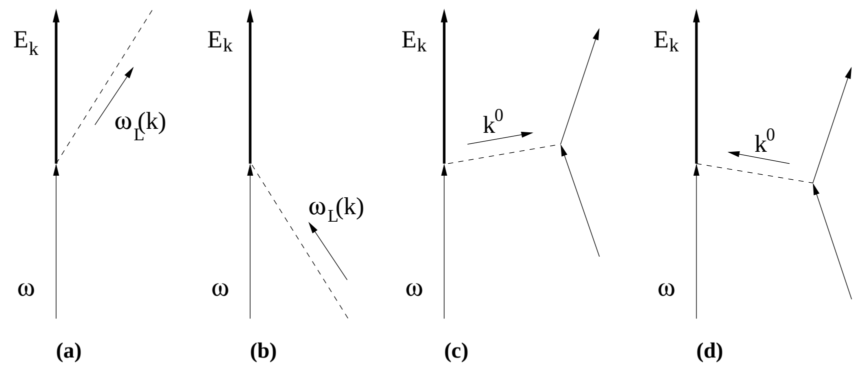
The zero-momentum HQ self-energy reads:

$$\Sigma(p^0) = g^2 C_F \int \frac{d\boldsymbol{k}}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dk^0}{2\pi} \rho_L(k^0, k) \frac{1 + N(k^0) - n_F(E_k)}{p^0 - E_k - k^0}$$

Test-particle limit recovered setting $n_F(E_k) = 0$, which arises naturally in the regime $T/M \ll 1$

$$\Sigma^{\rm test}(p^0) = g^2 C_F \int \frac{d\boldsymbol{k}}{(2\pi)^3} \int_0^{+\infty} \frac{dk^0}{2\pi} \rho_L(k^0, k) \left[\frac{1 + N(k^0)}{p^0 - E_k - k^0} + \frac{N(k^0)}{p^0 - E_k + k^0} \right]$$

HQ self-energy: imaginary-part

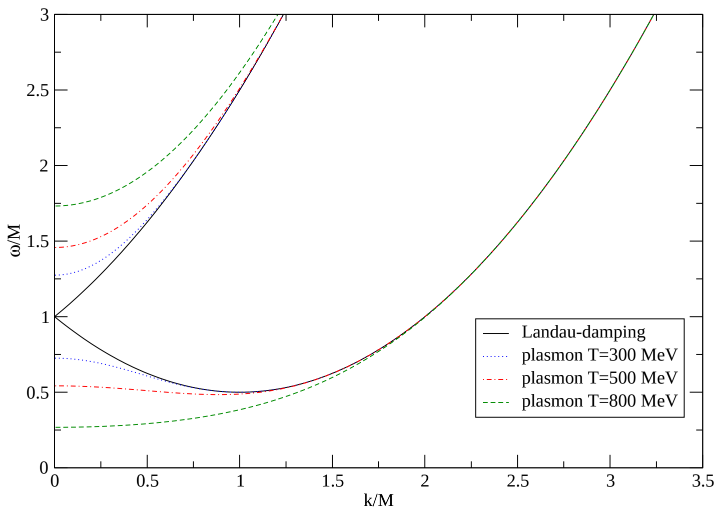


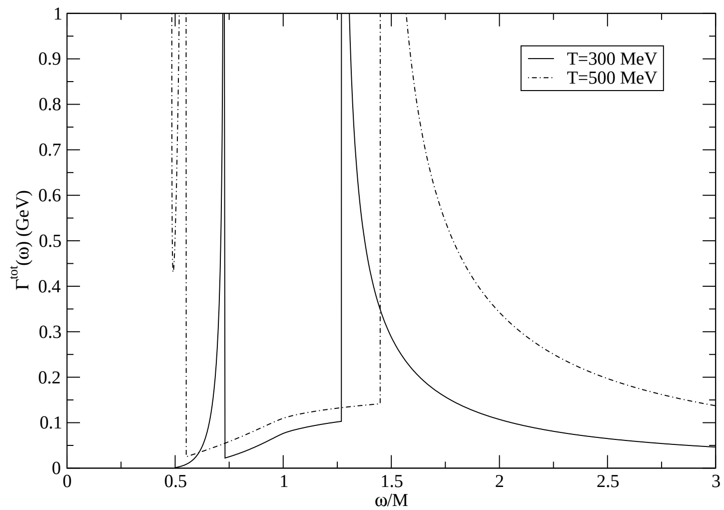
- Plasmon-pole contribution (a and b)

$$\Gamma^{\rm pole}(\omega)=g^2C_F\int\frac{d\boldsymbol{k}}{(2\pi)^3}(2\pi)Z_L(k)\times$$
$$\times\left[(1+N(\omega_L(k)))\,\delta(\omega-E_k-\omega_L(k))+N(\omega_L(k))\delta(\omega-E_k+\omega_L(k))\right]$$

- Continuum contribution (c and d)

$$\Gamma^{\rm cont}(\omega)=g^2C_F\int\frac{d\boldsymbol{k}}{(2\pi)^3}\int_0^k dk^0\,\beta_L(k^0,k)\times$$
$$\times\,(2\pi)\left\{\left[1+N(k^0)\right]\delta(\omega-E_k-k^0)+N(k^0)\delta(\omega-E_k+k^0)\right\}$$



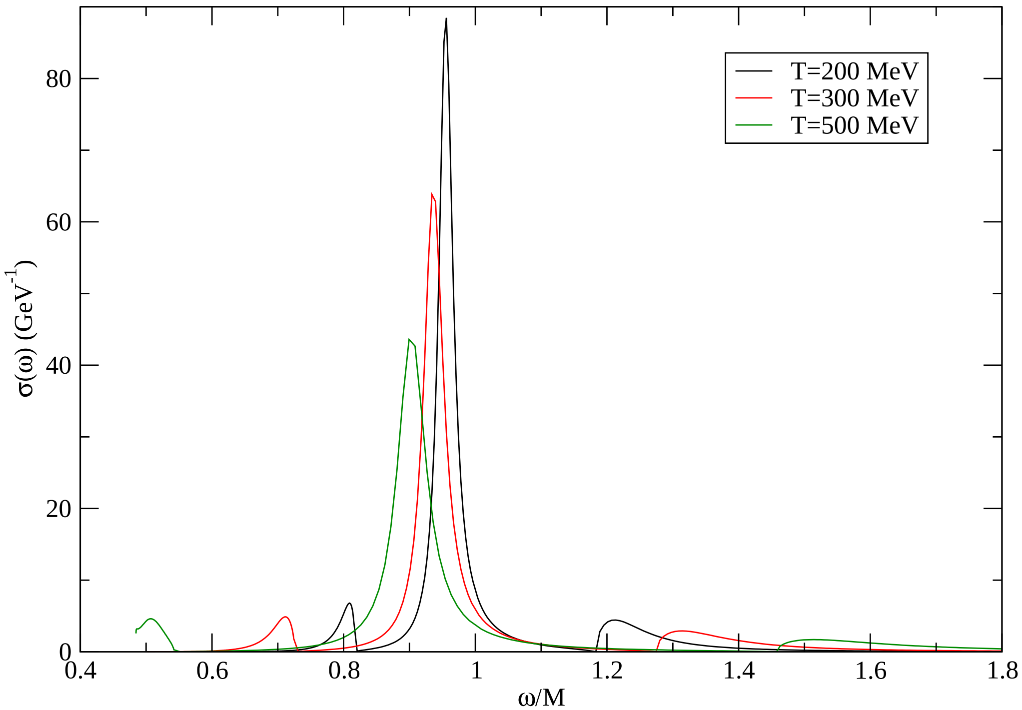


- Spectrum displaying a **threshold close to $M/2$**;
- Very narrow *peaks arising from a divergence in the density of states* (Van-Hove singularities). Defining $\omega \equiv E_{k_{1/2}} \pm \omega_L(k_{1/2})$

$$\Gamma^{\text{pole}}(\omega) = \frac{g^2 C_F}{\pi} \left\{ \frac{k_1^2}{|E'_{k_1} + \omega'_L(k_1)|} Z_L(k_1) \left[1 + N(\omega_L(k_1))\right] + \right.$$

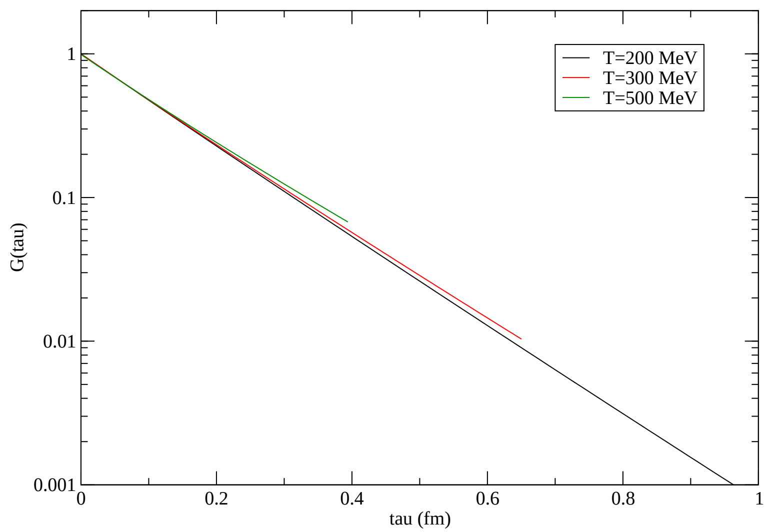
$$\left. + \sum_{k_2} \frac{k_2^2}{|E'_{k_2} - \omega'_L(k_2)|} Z_L(k_2) N(\omega_L(k_2)) \right\}$$

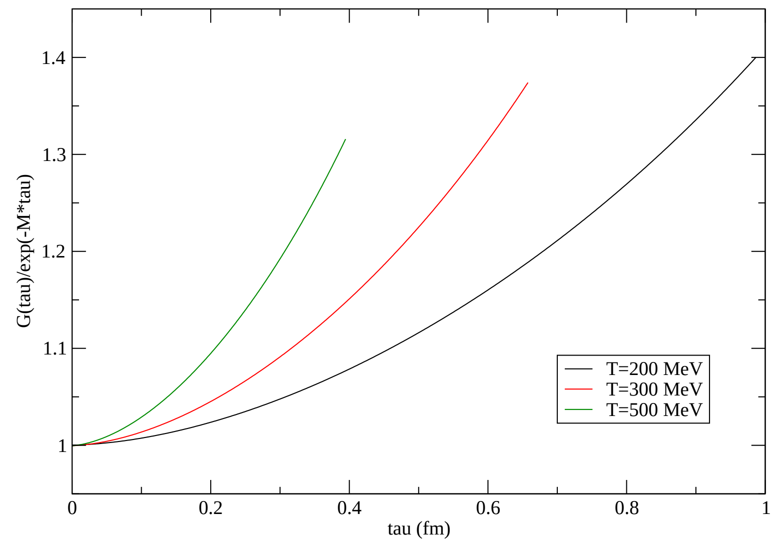
HQ spectral-function



- Negative shift and broadening of the principal peak;
- Appearance of secondary peaks at energies corresponding to a large density of states for *plasmon absorption/emission processes*

HQ euclidea correlator I





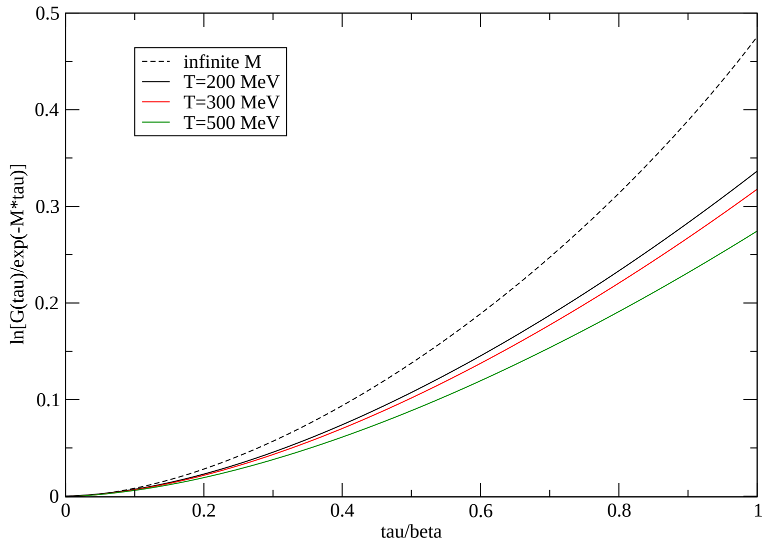
- Obtained (numerically) from

$$G(\tau) = \int \frac{d\omega}{2\pi}\, e^{-\omega\tau} \sigma(\omega);$$

- Its short-time behavior fullfills the sum-rules

$$G(0) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \sigma(\omega) = 1, \quad \text{and} \quad -G'(0) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, \omega\, \sigma(\omega) = M.$$

HQ euclidea correlator II



- Deviations from the (universal) $M = \infty$ curve get larger as T/M increases;
- Magnitude of free-energy shift

$$\exp[-\beta\, \Delta F_{Q,\boldsymbol{p}}] = G(\beta, \boldsymbol{p})$$

smaller than in the static case. *This in agreement with the shift of the main peak in the spectral density.*

Summary

- The effective-action approach, introduced to derive a real-time *static potential*, results very convenient to address also the finite-mass case: *QFT problem reduced to a QM problem!*
- Numerical results for $G(\tau)$ indicates the possible existence of secondary peaks and an important spectral strength at low-energy;
- Resummed one-loop calculation of interest to shed light on possible processes responsible for such a strength.

Future developments

- Systematic study for different values of the HQ mass, the temperature and the coupling;
- *Addressing the $Q\overline{Q}$ case*.

Back-up slides

Evaluation of the path-integral I

We can reduce

$$
\begin{aligned}
G(\tau, r) \quad &= \quad \int [\mathcal{D}\boldsymbol{z}] \exp\left[-\int_0^\tau d\tau' \left(M + \frac{1}{2} M \dot{\boldsymbol{z}}^2\right)\right] \times \\
&\quad \times \exp\left[\frac{g^2}{2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta_L^T(\tau' - \tau'', \boldsymbol{z}(\tau') - \boldsymbol{z}(\tau''))\right] \\
&\equiv \quad \int [\mathcal{D}\boldsymbol{z}] \exp[-S[\boldsymbol{z}]]
\end{aligned}
$$

to the evaluation of an *expectation value*, by *rescaling the coupling* $g^2 \to \alpha g^2$

$$
G_\alpha(\tau, r) \equiv \int [\mathcal{D}\boldsymbol{z}] \exp[-S_\alpha[\boldsymbol{z}]],
$$

so that

$$
\frac{\partial \ln G_\alpha(\tau, r)}{\partial \alpha} = \left\langle \frac{g^2}{2} \int d\tau' \int d\tau'' \Delta_L^T(\tau' - \tau'', \boldsymbol{z}(\tau') - \boldsymbol{z}(\tau'')) \right\rangle_\alpha
$$

Evaluation of the path-integral II

- For a given α the *expectation value* is evaluated by generating paths distributed according to

$$W_\alpha[\boldsymbol{z}] = \frac{1}{G_\alpha} \exp(-S_\alpha[\boldsymbol{z}])$$

- By integrating over the parameter α one gets:

$$\int_0^1 d\alpha \, \frac{\partial \ln G_\alpha(\tau, r)}{\partial \alpha} = \ln \left(\frac{G(\tau, r)}{G_{\text{free}}(\tau, r)} \right) = \int_0^1 d\alpha \, \langle \Delta \rangle_\alpha,$$

where

$$G_{\text{free}} = [M/(2\pi\tau)]^{3/2} \exp[-Mr^2/(2\tau)].$$

Renormalization of the path-integral correlator

In the path-integral correlator

$$G_{\mathrm{MC}}(\tau,\boldsymbol{r}) = \int_{\boldsymbol{z}(0)=\mathbf{0}}^{\boldsymbol{z}(\tau)=\boldsymbol{r}} [\mathcal{D}\boldsymbol{z}] \exp\left[-\int_0^\tau d\tau' \left(M + \frac{1}{2} M \dot{\boldsymbol{z}}^2\right)\right] \times$$

$$\times \exp\left[\frac{g^2}{2}\int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta_L^T(\tau'-\tau'', \boldsymbol{z}(\tau') - \boldsymbol{z}(\tau''))\right],$$

the interaction term is evaluated as:

$$\exp\left[\frac{g^2}{2}\sum_{i\neq j=1}^{\tau/a_t} a_t^2 \Delta_L^T(i-j, \boldsymbol{r}_i - \boldsymbol{r}_j)\right]$$

neglecting the $i=j$ contribution:

$$\exp\left[\frac{g^2}{2}\sum_{i=1}^{\tau/a_t} a_t^2 \Delta_L^T(0,\mathbf{0})\right] = \exp\left[\frac{g^2}{2} a_t \Delta_L^T(0,\mathbf{0})\,\tau\right]$$

- The finite time-step a_t provides a cutoff which insures dealing always with finite quantities in the intermediate steps;
- however we don't want to change the continuum physics.

The link with the continuum renormalized result is:

$$G_{\rm ren}(\tau,\boldsymbol{r}) = [Z(a_t)]^{\frac{\tau}{a_t}} G_{\rm MC}(\tau,\boldsymbol{r}|a_t).$$

The renormalization factor $Z(a_t)$ can be determined in the static case:

$$\overline{G}^{M=\infty}_{\rm ren}(\tau,\boldsymbol{r}=\boldsymbol{0}) = \exp\left\{\frac{g^2}{2}\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(\frac{1}{\boldsymbol{q}^2}-\frac{1}{\boldsymbol{q}^2+m_D^2}\right)\tau\right\}\times$$
$$\times\exp\left\{g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\int_0^{+\infty}\frac{dq^0}{2\pi}\frac{\rho_L(q^0,\boldsymbol{q})}{(q^0)^2}\left[\frac{\cosh q^0(\tau-\beta/2)}{\sinh(\beta q^0/2)}-\coth(\beta q^0/2)\right]\right\},$$

$$\overline{G}^{M=\infty}_{\rm MC}(\tau,\boldsymbol{r}=\boldsymbol{0}) = \exp\left[\frac{g^2}{2}\sum_{i\neq j=1}^{\tau/a_t} a_t^2\Delta_L^T(i-j,\boldsymbol{r}=\boldsymbol{0})\right].$$